Type II and Type III Compensators

1. Type II Compensator Using Op-Amp:

Offering an origin pole, one zero, and one high-frequency pole, the Type II compensator provides a phase boost up to 90 degrees. Figure 3 shows the electrical configuration, and the transfer function is obtained by calculating the impedance offered by the network placed in the Op-Amp feedback path (Z_f) and dividing it by the upper resistor (R_1).

$$H(s) = \frac{Z_f}{Z_i} = \frac{\left(\frac{1}{sC_1}+R_2\right)\frac{1}{sC_3} \Big/ \left(\left(\frac{1}{sC_1}+R_2\right)+\frac{1}{sC_3}\right)}{R_1} \tag{3}$$

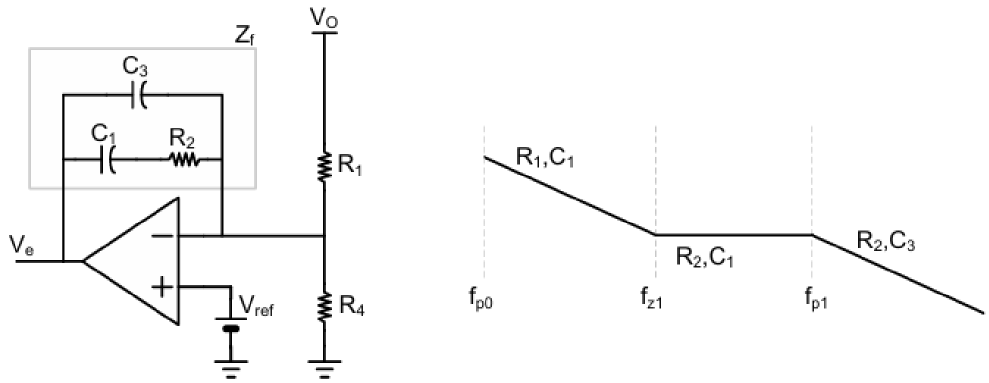


Figure 3. Type II Compensator with Gain Curve of Op-Amp

Rearranging equation (3) leads us to the transfer function we are looking for:

$$H(s) = \frac{1+C_1R_2s}{(C_3+C_1)R_1s+R_1C_1C_3R_2s^2} \tag{4}$$

Note that four components (R_1, R_2, C_1, and C_3) are involved in determining the poles and zero, and the locations of the poles and zero are:

$$f_{p0} = \tag{5}$$

$$f_{p1} = \frac{1}{2\pi\times R_2\times C_3} \tag{6}$$

$$f_{z1} = \frac{1}{2\pi\times R_2\times C_1} \tag{7}$$

We can find the required C_1, R_2, and C_3 once we select R_1 with the desired f_{p0}, f_{p1} and f_{z1}.

$$C_1 = \frac{1}{2\pi\times R_1\times f_{p0}} \tag{8}$$

$$R_2 = \frac{f_{p0}\times R_1}{f_{z1}} \tag{9}$$

$$C_3 = \frac{f_{z1}}{2\pi\times R_1\times f_{p0}\times f_{p1}} \tag{10}$$

Based on the locations of the poles and zero, Figure 4 shows an example for the compensator which has an appropriate shape, and usually a good phase margin.

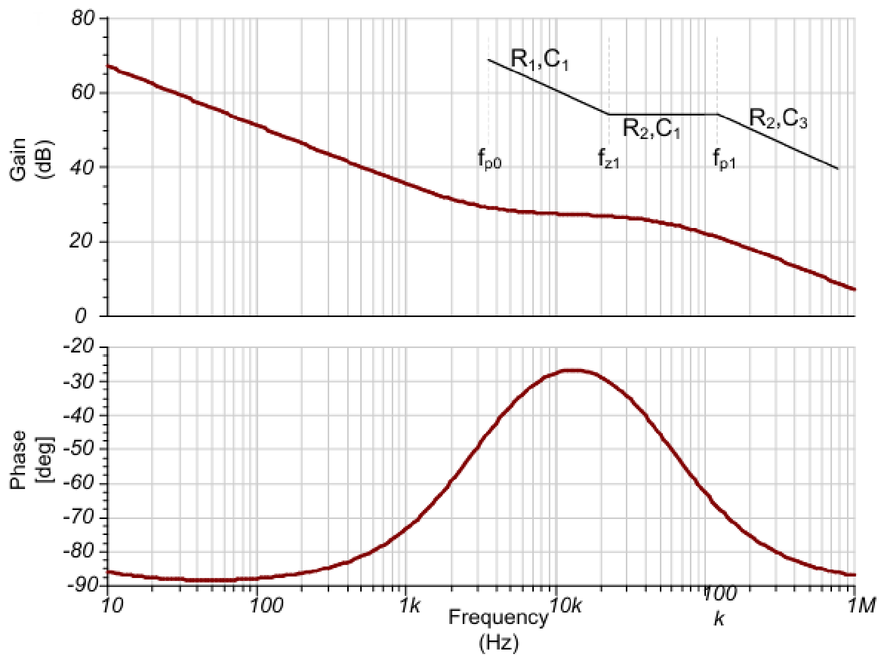


Figure 4. Appropriate Shape of Type II Compensator

A Type II compensation amplifier adds an RC branch to flatten the gain, and improve the phase response in the mid-frequency range. The increased phase is achieved by increasing the separation of the pole and zero of the compensation.

Note that this type of compensator always has a net negative phase, and it cannot be used to improve the phase of the power stage. For this reason, Type II compensators cannot be used for voltage-mode control in CCM where there is a large phase drop just after the resonant frequency. Type II compensators are usually reserved for current-mode control compensation, or for converters that always operate in the DCM region.

2. Type II Compensator Using OTA:

We can visualize this feedback stage as a product of three cascade transfer functions, H1(s), H2(s), and H3(s) as shown in Figure 5. It combines a pole/zero pair plus an origin pole for a high DC gain, and the transfer function is defined as:

$$H(s) = \frac{V_e(s)}{V_o(s)} = H1(s) \times H2(s) \times H3(s) \tag{11}$$

So, the transfer function we are looking for:

$$H(s) = -\frac{R_4}{R_1+R_4} \times g_m \times \frac{1+R_2C_1s}{(C_3+C_1)s+R_2C_3C_1s^2} \tag{12}$$

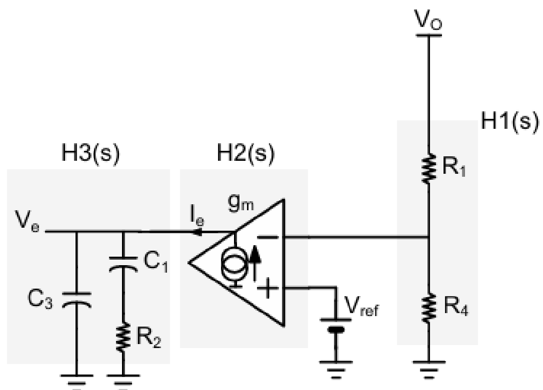


Figure 5. Type II Compensator with OTA

If we factor sR_2C_1, we can easily unveil the poles and zeros definitions as follows:

$$H(s) = -\frac{R_4 g_m}{R_1+R_4} \times \frac{R_2C_1}{C_1+C_3} \times \frac{1+\frac{1}{sR_2C_1}}{1+sR_2\frac{C_1C_3}{C_1+C_3}} = -G_0\frac{1+{}^{\omega_z}/_{s}}{1+{}^{s}/_{\omega_p}} \tag{13}$$

In this equation, we can identify the three pertinent terms:

$$G_0 = \frac{R_4}{R_1+R_4}\,\frac{g_mR_2C_1}{C_1+C_3} \tag{14}$$

$$f_z = \frac{1}{2\pi R_2C_1} \tag{15}$$

$$f_p = \frac{C_1+C_3}{2\pi R_2C_1C_3} \tag{16}$$

To extract each individual value, we have to derive the magnitude of equation (13) at the crossover frequency (f_c):

$$|G(f_c)| = G_0\frac{\sqrt{1+(\frac{f_z}{f_c})^2}}{\sqrt{1+(\frac{f_c}{f_p})^2}} \tag{17}$$

Combining (14), (15), (16), and (17), we find the following external components:

$$R_2 = \frac{f_p G(R_4+R_1)}{(f_p-f_z)R_4 g_m} \times \frac{a}{b} \tag{18}$$

$$C_1 = \frac{1}{2\pi f_z R_2} \tag{19}$$

$$C_3 = \frac{R_4 g_m}{2\pi f_p G(R_4+R_1)} \times \frac{b}{a} \tag{20}$$

Where:

$$a = \sqrt{1 + (\frac{f_c}{f_p})^2} \tag{21}$$

$$b = \sqrt{1 + (\frac{f_z}{f_c})^2} \tag{22}$$

G represents the gain or attenuation you are looking for at the selected crossover frequency (f_c). As a design example, if we need a 25 dB attenuation at 10 kHz with a phase boost of 50 degrees, selecting 100 µs transconductance of OTA (also, R1 = 40 kΩ, R4 = 25 kΩ), the external components (R_2, C_1 and C_3) are calculated as follows:

$$f_p = \left(tan\left(boost \times \frac{\pi}{180}\right) + \sqrt{tan\left(boost \times \frac{\pi}{180}\right)^2 + 1}\right) \times f_c = 27.5\, kHz \tag{23}$$

As the phase peaks at the geometric mean between the pole and the zero, this letter is placed at:

$$f_z = \frac{{f_c}^2}{f_p} = 3.64\, kHz \tag{24}$$

Applying the design definition from (18) through (22), we find

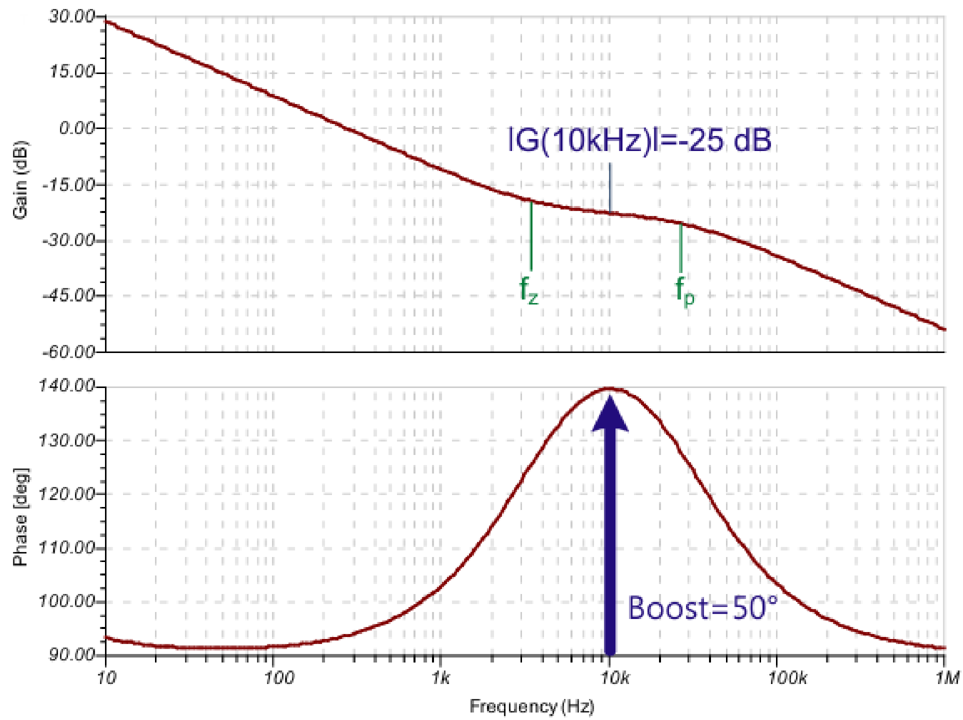
$$G = 10^{\frac{G_{fc}}{20}} = 10^{\frac{-25}{20}} = 0.056 \tag{25}$$

$$R_2 = \frac{27.5k \times 0.056(40k+25k)}{(27.5k-3.64k)25k \times 100\mu} \times \frac{\sqrt{(\frac{10k}{27.5k})^2+1}}{\sqrt{(\frac{3.64k}{10k})^2+1}} = 1.685\, k\Omega \tag{26}$$

$$C_1 = \frac{1}{2 \times 3.14 \times 3.64k \times 1.685k} = 25.95\, nF \tag{27}$$

$$C_3 = \frac{25k \times 100\mu}{2 \times 3.14 \times 27.5k \times 0.056(40k+25k)} \times \frac{\sqrt{(\frac{3.64k}{10k})^2+1}}{\sqrt{(\frac{10k}{27.5k})^2+1}} = 3.96\, nF \tag{28}$$

The results of this simulation with the given parameters appear in Figure 6 and show the –25 dB transition occurring at 10 kHz, as expected. Also, the boost in phase is 50 degrees as required.



3. Type III Compensator Using Op-Amp:

Figure 7 shows the conventional Type III compensation using voltage Op-Amp. There are two poles (f_{p1} and f_{p2}, besides the pole-at-zero f_{p0}) and two zeros (f_{z1} and f_{z2}) provided by this compensation. The Type III compensator is used when more than 90 degrees of phase boost are necessary. By adding another pole/zero pair to the Type II compensator, the Type III can theoretically boost the phase up to 180 degrees. The derivation of its transfer function does not really change, which means the principle remains the same with the Type II method.

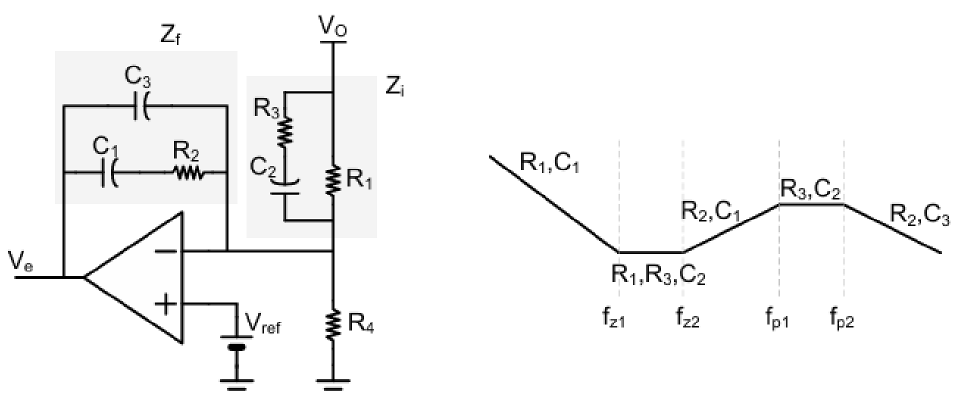


Figure 7. Type III Compensator with Gain Curve

We need to calculate the equivalent impedance Z_f placed across the Op-Amp, and divide it by that of Z_i to further rearrange the equation:

$$H(s) = \frac{Z_f}{Z_i} = \frac{\left(\frac{1}{sC_1}+R_2\right)\frac{1}{sC_3} \Big/ \left(\left(\frac{1}{sC_1}+R_2\right)+\frac{1}{sC_3}\right)}{\left(\frac{1}{sC_2}+R_1\right)R_1 \Big/ \left(\left(\frac{1}{sC_2}+R_3\right)+R_1\right)} \tag{29}$$

We obtain a more familiar expression:

$$H(s) = \frac{(sC_2(R_1+R_3)+1)(sC_1R_2+1)}{\left(sR_1(C_1+C_3)\right)(sC_2R_3+1)(s\frac{C_1C_3R_2}{C_1+C_3}+1)} \approx \frac{(sC_2(R_1+R_3)+1)(sC_1R_2+1)}{(sR_1C_1)(sC_2R_3+1)(sR_2C_3+1)}, \quad \textit{if } C_1 >> C_3 \tag{30}$$

Note that several of the components involved play a dual role in determining the poles and zeros. So, the calculation can become fairly cumbersome and iterative. But a valid simplifying assumption that can be made is that C_1 is much greater than C_3. So the locations of the poles and zeros are finally:

$$f_{p0} = \frac{1}{2\pi \times R_1(C_1+C_3)} \approx \frac{1}{2\pi \times R_1C_1} \tag{31}$$

$$f_{p1} = \frac{1}{2\pi \times R_3C_2} \tag{32}$$

$$f_{p2} = \frac{1}{2\pi \times R_2\left(C_1C_3 / C_1+C_3\right)} = \frac{1}{2\pi \times R_2}\left(\frac{1}{C_1}+\frac{1}{C_3}\right) \approx \frac{1}{2\pi \times R_2C_3} \tag{33}$$

$$f_{z1} = \frac{1}{2\pi \times (R_1+R_3)C_2} \tag{34}$$

$$f_{z2} = \frac{1}{2\pi \times R_2C_1} \tag{35}$$

We can find the required C_1, C_2, C_3, R_2, and R_3 once we select R_1 with the desired f_{p0}, f_{p1}, f_{p2}, $f_{z1,}$ and f_{z2} as follows.

$$C_1 = \frac{f_{p2}-f_{z2}}{2\pi \times R_1f_{p0}f_{p2}} \tag{36}$$

$$C_2 = \frac{f_{p1}-f_{z1}}{2\pi \times R_1f_{p1}f_{z1}} \tag{37}$$

$$C_3 = \frac{f_{z2}}{2\pi \times R_1f_{p0}f_{p2}} \tag{38}$$

$$R_2 = \frac{R_1f_{p0}f_{p2}}{(f_{p2}-f_{z2})f_{z2}} \tag{39}$$

$$R_3 = \frac{R_1f_{z1}}{f_{p1}-f_{z1}} \tag{40}$$

Based on the locations of the poles and zeros, Figure 8 shows a simulation example for the compensator which has an appropriate shape. This compensation scheme adds another RC branch to the Type II compensator. There is an integrator at low frequencies, followed by a pair of zeros. After this, the compensator has a region during where the gain increases with frequency, and the phase during this region is positive. In effect, the compensator is performing the function of a differentiator, measuring the slope of the power supply output waveform.

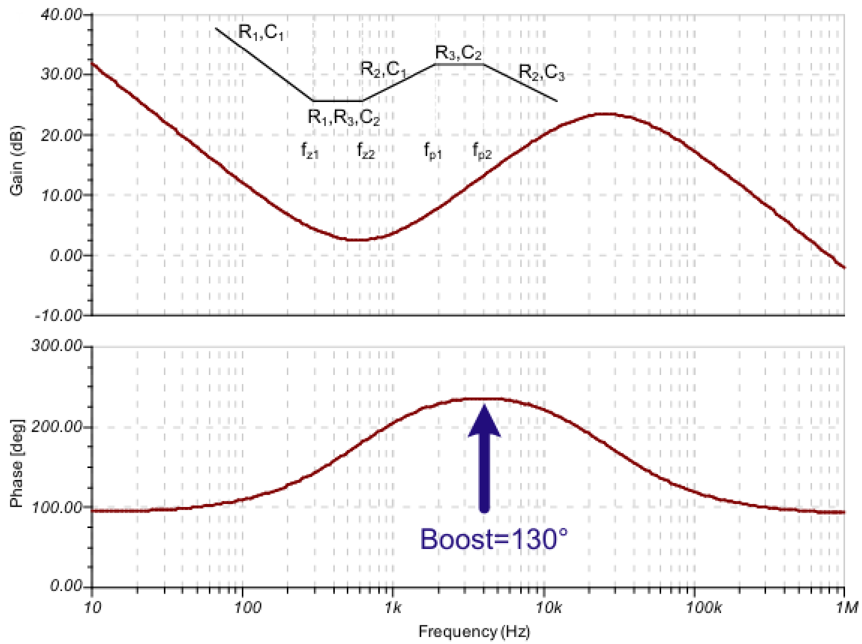


Figure 8. AC Simulation Results of Type III Op-Amp Compensator

The phase boost in this case is about 130 degrees, but this can be increased by further separation of the poles and zeros. However, you cannot arbitrarily increase this pole-zero separation without considering the detrimental effects on shape of the loop gain. The Type III amplifier is the one for the voltage-mode converters operating in CCM.

4. Type III Compensator Using OTA:

By combining an RC network across the upper resistor R_1, it is possible to build a Type III with an OTA. We can also visualize this feedback stage as a product of three cascade transfer functions, H1(s), H2(s), and H3(s) as shown in Figure 9. The transfer function is defined as:

$$H(s) = \frac{V_e(s)}{V_o(s)} = H1(s) \times H2(s) \times H3(s) \tag{41}$$

We know that the output current of the OTA depends on the voltage difference developed between its inputs. In AC, the voltage on the inverting pin depends on the divider network. So, the transfer function we are looking for:

$$H(s) = -\frac{R_4+(R_1+R_3)C_2R_4s}{R_1+R_4+(R_4R_1+R_3R_1+R_3R_4)C_2s} \times g_m \times \frac{1+R_2C_1s}{(C_3+C_1)s+R_2C_3C_1s^2} \tag{42}$$

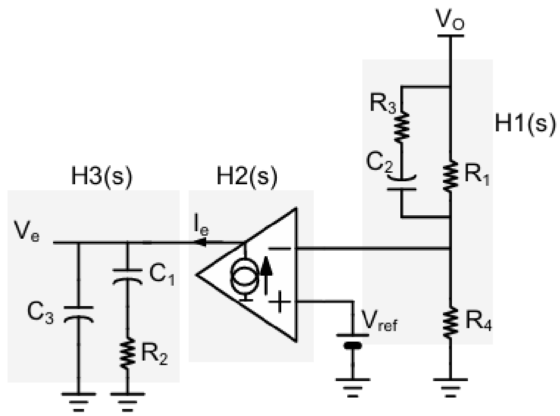


Figure 9. Type III Compensator with OTA

Developing and arranging equation (42) leads to:

$$H(s) = -\frac{R_4g_m}{R_4+R_1} \times \frac{sC_2(R_3+R_1)+1}{\left(sC_2\left(\frac{R_4R_1}{R_4+R_1}+R_3\right)+1\right)} \times \frac{1+sR_2C_1}{\left(s(C_1+C_2)\left(1+sR_2\frac{C_1C_3}{C_1+C_3}\right)\right)} \tag{43}$$

Factoring $1+sR_2C_1$, we have:

$$H(s) = -\frac{R_4g_mR_2C_1}{(R_4+R_1)(C_1+C_3)} \times \frac{1+\frac{1}{sR_2C_1}}{\left(sC_2\left(\frac{R_4R_1}{R_4+R_1}+R_3\right)+1\right)} \times \frac{sC_2(R_3+R_1)+1}{\left(1+sR_2\frac{C_1C_3}{C_1+C_3}\right)} \tag{44}$$

This expression can be put in the normalized form:

$$H(s) = -G_0 \times \frac{\left(1+\frac{\omega_{z1}}{s}\right)\left(1+\frac{s}{\omega_{z2}}\right)}{\left(1+\frac{s}{\omega_{p1}}\right)\left(1+\frac{s}{\omega_{p2}}\right)} \tag{45}$$

where:

$$G_0 = \frac{R_4g_mR_2C_1}{(R_4+R_1)(C_1+C_3)} \tag{46}$$

$$f_{z1} = \frac{1}{2\pi R_2C_1} \tag{47}$$

$$f_{z2} = \frac{1}{2\pi(R_1+R_3)C_2} \tag{48}$$

$$f_{p1} = \frac{(C_1+C_3)}{2\pi R_2 C_1 C_3} \tag{49}$$

$$f_{p2} = \frac{1}{2\pi(\frac{R_4 R_1}{R_4+R_1}+R_3)C_2} \tag{50}$$

From the values, we need to extract the design expression that will let us calculate the passive values. We first need the magnitude of G at the selected crossover frequency:

$$|G(f_c)| = G_0 \times \frac{c \times d}{a \times b} \tag{51}$$

where:

$$a = \sqrt{(\frac{f_c}{f_{p2}})^2 + 1} \tag{52}$$

$$b = \sqrt{(\frac{f_c}{f_{p1}})^2 + 1} \tag{53}$$

$$c = \sqrt{(\frac{f_{z1}}{f_c})^2 + 1} \tag{54}$$

$$d = \sqrt{(\frac{f_c}{f_{z2}})^2 + 1} \tag{55}$$

$$aa = \frac{a \times b}{c \times d} \tag{56}$$

$$bb = \frac{G f_{p2}(R_1+R_4)}{R_4 g_m(f_{p2}-f_{z1})} \tag{57}$$

$$cc = \left|R_1^2 f_{z2} - R_1 R_4 (f_{p1} - f_{z2})\right| \tag{58}$$

$$dd = (f_{p1} - f_{z2})(R_1 + R_4) \tag{59}$$

By using equations from (47) to (59), we can solve for R_2, R_3, C_1, C_2, and C_3:

$$R_2 = aa \times bb \tag{60}$$

$$R_3 = \frac{cc}{dd} \tag{61}$$

$$C_1 = \frac{1}{2\pi f_{z1} R_2} \tag{62}$$

$$C_2 = \frac{1}{2\pi f_{z2}(R_3+R_1)} \tag{63}$$

$$C_3 = \frac{C_1}{2\pi C_1 R_2 f_{p2} - 1} \tag{64}$$